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Giant gravitons in AdS/CFT
1. Matrix model and back reaction

Marco M. Caldarelli and Pedro J. Silva

Dipartimento di Fisica dell'Università di Milano, and
INFN, Sezione di Milano
Via Celoria 16, I-20133 Milano, Italy
E-mail: marco.caldarelli@mi.infn.it, pedro.silva@mi.infn.it

Abstract: In this article we study giant gravitons in the framework of AdS/CFT correspondence. First, we show how to describe these configurations in the CFT side using a matrix model. In this picture, giant gravitons are realized as single excitations high above a Fermi sea, or as deep holes into it. Then, we give a prescription to define quasi-classical states and we recover the known classical solution associated to the CFT dual of a giant graviton that grows in AdS. Second, we use the AdS/CFT dictionary to obtain the supergravity boundary stress tensor of a general state and to holographically reconstruct the bulk metric, obtaining the back reaction of space-time. We find that the space-time response to all the supersymmetric giant graviton states is of the same form, producing the singular BPS limit of the three charge Reissner-Nordström-AdS black holes. While computing the boundary stress tensor, we comment on the finite counterterm recently introduced by Liu and Sabra, and connect it to a scheme-dependent conformal anomaly.

Keywords: D-branes, AdS-CFT and dS-CFT Correspondence, Matrix Models.

Contents

1. Introduction

In the latest time the AdS/CFT conjecture has been one of the most studied subjects within string theory. This duality gives the possibility to study properties of supergravity theories from the CFT point of view and vice versa. In particular it has brought new insights into black hole physics and the role of naked singularities in AdS (see [1, 2, 3] for reviews).

Although the initial studies in AdS/CFT where focused on the supergravity approximation, lately the inclusion of different D-brane configurations has enriched our understanding, exploring new sectors that are necessary for the consistency of the whole picture. In particular, since the AdS_5/CFT_4 duality was constructed using D3-branes, stable D3-brane configurations in AdS play an important role among all the different D-brane sectors.

In this work we focus on a special type of D3-brane configurations called giant gravitons (GGs). These GGs are stabilized by a dynamical mechanism that develops local forces on the brane, canceling its tension and avoiding therefore the worldvolume collapse. Originally, GGs were described as D3-branes traveling on a $\mathcal{S}^1$ direction wrapping a perpendicular $\mathcal{S}^3$, both contained in the $\mathcal{S}^5$ factor of the metric, while they sit on the center of AdS_5 [4].[1] The dynamics of the D3-brane effective action allows for two different stable solutions, one

[1]Nowadays we count with different generalization to this initial solution, where GGs travel along generic geodesics on AdS_5 [5], and where the D3-brane wraps general 3-cycles on the $\mathcal{S}^5$ [6]. For a non-supersymmetric extension, see [7].

in which the radius of the $\mathcal{S}^3$ is zero and the other with a non-vanishing radius, bounded from above and proportional to the momentum along the $\mathcal{S}^1$ direction. Short after these configurations were studied, another solution was found, where this time the D3-brane wraps an $\mathcal{S}^3$ inside the AdS_5 part of the ten-dimensional space-time [8]. All these three solutions share the same quantum numbers and charges. In particular, they preserve half of the supersymmetries [9] and, from the ten-dimensional point of view, their geometrical center travels along a null geodesic.

In this article, the first type of configuration will be called *GG in* $\mathcal{S}^5$ while the second kind will be called *GG in* AdS_5. The point-like configuration corresponds to a degenerate solution of the Born-Infeld action and will not be considered here. Originally, these configurations where thought as the gravitational manifestation of the stringy exclusion principle [10], where the upper bound on the giant graviton momentum on the $\mathcal{S}^1$ (due to the fact that it is proportional to the radius of the $\mathcal{S}^3$ and therefore has a maximum on $\mathcal{S}^5$), is dual to the upper bound found on the conformal weight of a family of chiral operators (here, the bound is easily understood from the finite rank of the gauge symmetry group $\mathrm{U}(N)$) [4, 11]. But, unfortunately, the second kind of configuration, the GG in AdS, has a completely different behavior, with no such upper bound. This fact, added to the existence of the singular solution, shows a not so clear picture (see [12] for a discussion of this point).

GGs have also been studied from the dual CFT point of view. In [11], Balasubramanian et al. proposed a particular class of operators called sub-determinants as the duals of GGs in $\mathcal{S}^5$. Later on, Corley et al. [13] extended this proposition to describe all GGs in terms of a particular combination of single trace and multi-trace operators on the real scalars of the CFT theory, known as Schur polynomials. On the top of the above picture, GGs in AdS also have a dual description in terms of a semi-classical solution of the CFT theory [8], that remains to be connected to the quantum theory (such links are conjectured in [13, 14]). These semi-classical states provide a picture where GGs may be seen as single D3-branes separating from the initial stack of N D3-branes.

In this work, our first purpose is to define a theoretical framework simple enough to be manageable, but containing the principal physical ingredients that characterize these D3-brane states. To this end, we use a few basic assumptions to reduce the sector of the CFT theory relevant for the discussion of GGs to a quantum matrix model. Then, following the ideas of Hashimoto et al. [8], Corley et al. [13] and Berestein [14], we arrive to a compact and elegant description of the quantum system in terms fermionic degrees of freedom. At this point, GG states are identified with Schur polynomials acting on a Fermi sea. This procedure leads to a beautiful interpretation of GGs as either holes in the deep of the Fermi sea or as highly excited states over the surface. Then, we identify quasi-classical states and, as a particular case, recover the classical solution of [8].

Then, we proceed to our second purpose, which is to obtain the back reaction on the supergravity fields due to the presence of GGs in $AdS\times\mathcal{S}^5$ *space-time.* To this end, we reconstruct the supergravity solution from the boundary data using the AdS/CFT dictionary, and deduce that, at least asymptotically, the GG back reaction produces the singular BPS limit of the Reissner-Nordström-AdS_5 family of solutions, with one or more R-charges

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turned on [15]. In the quasi-classical limit the branes localize in space-time and therefore the fields they source are reproduced accurately by the supergravity solution we obtain, up to regions of high curvature near the singularity. This is where the GG condensate lies and supergravity looses its validity.

These supergravity solutions were assigned originally to a GG condensate in $\mathcal{S}^5$ in [16] by a different argument, but from our analysis GGs in $\mathcal{S}^5$ and GGs in AdS$_5$ produce the same back reaction. This may seem surprising at first sight. In fact, supergravity does not seem to be able to distinguish between different GGs configurations with the same quantum numbers. Nevertheless, we argue that the back reaction solution has different ranges of validity for the two types of configurations and it makes sense to trust the supergravity solution deep into the bulk only when considering GGs in AdS.

To compare the supergravity and CFT stress energy tensors, we need to compute the boundary stress tensor for the three-charge family of Reissner-Nordström-AdS$_5$ solutions. In order to obtain the results with the same renormalization scheme for the two sides of the correspondence, we have to add precisely the finite counterterm recently introduced by Liu and Sabra [17] (see [18] for previous works on the subject). Finally, we argue that this boundary term is exactly the one needed to cancel the scheme-dependent contribution to the conformal anomaly of the CFT.

The plan of the work is the following: in section 2, we introduce the CFT notations and conventions to define the dual operators of GGs. Then, we describe the reduction of the CFT system that leads us to a quantum mechanical matrix model and to the description of GGs on it, including quasi-classical states. In section 3, we compute the expectation value of the quantum stress energy tensor of the GG states, and match it with the appropriate five dimensional supergravity solution. At last, in section 4 we summarize and discuss our results. The details of the calculations are left to the appendices. In appendix A we give and justify the definition of coherent and quasi-classical states for the reduced CFT model, while in appendix B we perform the computation of the boundary stress tensor and discuss the finite counterterm introduced by Liu and Sabra.

2. Matrix model for giant gravitons

Giant gravitons have been identified with a particular class of half-BPS operators made out of the real scalars X^m of $\mathcal{N} = 4$ SYM theory.[2] We use in this article the $\mathcal{N} = 1$ decomposition, where the scalars are usually written as three complex scalars $\Phi^I = \frac{1}{\sqrt{2}}(X^I + iX^{I+3})$, with $I = 1, 2, 3$, and all the fields transform in the adjoint representation of U(N). GGs are a combination of single-trace and multi-trace operators in Φ^I, labeled by their R-charge n. These operators are then identified with Schur polynomial in Φ^I, written either in the totally symmetric representation U of the associated symmetric group

[2] In this article, we use greek indices $\mu, \nu, \ldots = 0 \ldots 5$ for the tangent space of five-dimensional space-time and latin indices $a, b, \ldots = 0, \ldots, 3$ for the four-dimensional space-time on which the CFT lives. The U(N) and SU(N) gauge group indices are $i, j, \ldots = 1, \ldots, N$ and for the R-charges we use $m, n, \ldots = 1, \ldots, 6$ indices when described in terms of the fundamental of SO(6) and capital $I, J, \ldots = 1, 2, 3$ indices when described in terms of the fundamental of SU(4).

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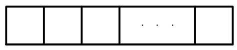
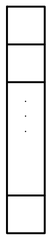
S_n (corresponding to a GG in AdS$_5$) or the totally antisymmetric representation U' of the symmetric group S_n (corresponding to a GG in $\mathcal{S}^5$) [13]. To be more precise, Schur polynomial operators are defined as

$$\chi_{(n,R)}\left(\Phi\right) = \frac{1}{n!} \sum_{\sigma \in S_n} \chi_R(\sigma)\, \Phi^{i_1}_{\sigma(i_1)} \cdots \Phi^{i_n}_{\sigma(i_n)} \tag{2.1}$$

where, without loss of generality, we have set the SU(4) indices I to 1 and will neglect it for the rest of this section. We also have written explicitly the U(N) indices 'i', taking values from 1 to N. The sum is over all the group elements σ of the symmetric group S_n and $\chi(\sigma)$ is the character of the element σ in the chosen representation R. The result of the permutation σ acting on the natural number 'i' is written as $\sigma(i)$. In particular, the U and U' representations of S_n have the following associated Young diagrams



U symmetric



U' antisymmetric

where the first is totally symmetric while the second is totally antisymmetric.[3] For example,

$$\chi_{(2,U)} = \frac{1}{2}\left[(\mathrm{Tr}\,\Phi)^2 + \mathrm{Tr}(\Phi^2)\right] \qquad \text{and} \qquad \chi_{(2,U')} = \frac{1}{2}\left[(\mathrm{Tr}\,\Phi)^2 - \mathrm{Tr}(\Phi^2)\right] \tag{2.2}$$

are respectively the Schur polynomials of degree $n = 2$ in the U and U' representations.[4]

2.1 Matrix model

In [14] it was pointed out that, in a particular regime, GGs could be described by a matrix model. Basically, the idea is to work in the frame where the CFT lives in a space-time with $\mathbb{R} \times \mathcal{S}^3$ topology, and then consider configurations homogeneous in the $\mathcal{S}^3$ (i.e. after expanding the CFT fields on spherical harmonics, we keep only the singlet states). Note that, the only relevant part of the gauge field $\mathbf{A}$ is the time-component, that we can always gauge away keeping in mind that the constraints tell us to consider only U(N)-invariant states. Effectively, we have reduced the system to a quantum mechanics since only time-dependence is allowed.[5]

[3] This diagrams are related to representations of S_n and should not be confused with diagrams related to representations of U(N).

[4] This is just an example to understand the structure of the Schur polynomial, and it must be remembered that we always work in the case where n is comparable to N.

[5] This is very similar to the approximation made when GGs are discussed in the test brane picture. There, the static gauge is implemented and the degrees of freedom corresponding to oscillations of the shape of the GG are frozen.

The relevant reduced action was considered in [8], and reads

$$S = \frac{\Omega_3 \ell^3}{g_{\rm YM}^2} \int dt \;\, \mathrm{Tr} \left(\dot{\Phi}^* \dot{\Phi} - \omega^2 \Phi^* \Phi \right) , \tag{2.3}$$

where $\omega = 1/\ell$ the inverse radius of AdS_5 and Ω_3 is the volume of the unit three sphere. Also, $*$ stands for complex conjugation. This action corresponds to the case where we have angular momentum only in the plane defined by (X^1, X^4). The general action, with angular momentum in all three planes, corresponds to three copies of the above action, one for each field Φ^1, Φ^2 and Φ^3.

Following the usual treatment,[6] we define two independent matrix valued oscillators A and B by $A = \frac{1}{\sqrt{2}}(\beta \Phi + \frac{i}{\beta} \Pi^\dagger)$ and $B = \frac{1}{\sqrt{2}}(\beta \Phi^\dagger + \frac{i}{\beta} \Pi)$ with $\beta = \sqrt{\Omega_3 \ell^2 / g_{\rm YM}^2}$, and the conjugate momenta $\Pi = \frac{\delta S}{\delta \dot{\Phi}}$. The hamiltonian H and angular momentum[7] J, in terms of A and B, become

$$H = \omega \; \mathrm{Tr} \left(A^\dagger A + B^\dagger B \right) , \qquad J = \mathrm{Tr} \left(A^\dagger A - B^\dagger B \right) , \tag{2.4}$$

and the only nontrivial commutation relations are

$$\left[A^i_j , (A^\dagger)^k_l \right] = \delta^k_j \delta^i_l \, , \qquad \left[B^i_j , (B^\dagger)^k_l \right] = \delta^k_j \delta^i_l \, . \tag{2.5}$$

It follows that A and B are lowering operators for the hamiltonian, and $A^\dagger$ and $B^\dagger$ are raising operators. As usual, the vacuum state $|0\rangle$ is defined to be annihilated by any matrix element of A and B. Generic states in the Hilbert space are then obtained from the vacuum by the repeated action of any matrix elements of $A^\dagger$ and $B^\dagger$. Moreover, the application of $A^\dagger$ raises the angular momentum by one, while $B^\dagger$ lowers it by one. Therefore, the matrix valued oscillators A and B are left and right handed oscillators respectively, exciting the two linear harmonic oscillators X^1 and X^4 with a phase shift of $\pm\pi/2$. Only the chiral states, obtained by the exclusive action of $A^\dagger$ (or $B^\dagger$) are supersymmetric, saturating the BPS bound $H = \omega J$. Here we will concentrate in the supersymmetric states obtained by the action of $A^\dagger$ alone.

The matrices A and B admit the so-called polar coordinates decomposition,[8] where we write $A = \Omega_A^\dagger \hat{A}\, \Omega_A$, $B = \Omega_B^\dagger \hat{B}\, \Omega_B$ with $\hat{A}$ and $\hat{B}$ diagonal matrices and Ω_A and Ω_B stand for the angular variables. Note that we still have the $\mathbb{Z}^N$ invariance acting on the diagonal matrices. Next, we can always diagonalize one of them, say A (but not B simultaneously), by acting with a similarity transformation of $\mathrm{U}(N)$. This transformation also changes the measure in the corresponding path integral of the quantum system. This change can be reabsorbed by a Vandermonde factor that multiplies the vacuum state. Since this factor is antisymmetric in the elements of $\hat{A}^\dagger$ and the hamiltonian still has the $\mathbb{Z}^N$ symmetry on these same elements, all the excited states will be antisymmetric in $\hat{A}^\dagger$, rendering our model effectively a fermionic system (again, we will consider only excitations made out of $A^\dagger$ to have BPS states).

[6] In [13] similar but different definitions are used.

[7] These operators J^I correspond to the angular momenta on the plane (X^I, X^{I+3}) and measure the R-charge of the states.

[8] See for example [19] for a review in matrix models in string theory and for references.

To be more concrete, we first write $\hat{A}^\dagger = \mathrm{diag}(a_1^\dagger, \ldots, a_N^\dagger)$, then the new normalized vacuum (or *fermionic vacuum*) is

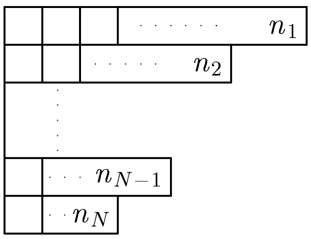
$$|f\rangle = \frac{1}{\sqrt{f_v}} \prod_{i<j} (a_i^\dagger - a_j^\dagger)\,|0\rangle\,, \qquad \text{with} \quad f_v = \prod_{k=1}^{N} k!\,, \tag{2.6}$$

where the operator in front of $|0\rangle$ is the normalized Vandermonde determinant. Generic BPS states are written as $S(a^\dagger)\,|f\rangle$, with $S(a^\dagger)$ symmetric in $a_i^\dagger$. Therefore, the overall functions acting on $|0\rangle$ are antisymmetric and can always be written as a Slater determinant of the following form

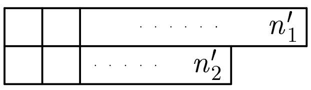
$$|\vec{n}\rangle = \det \begin{pmatrix} (a_1^\dagger)^{n_1} & (a_1^\dagger)^{n_2} & \cdots & (a_1^\dagger)^{n_N} \\ (a_2^\dagger)^{n_1} & (a_2^\dagger)^{n_2} & \cdots & (a_2^\dagger)^{n_N} \\ \vdots & \vdots & \ddots & \vdots \\ (a_N^\dagger)^{n_1} & (a_N^\dagger)^{n_2} & \cdots & (a_N^\dagger)^{n_N} \end{pmatrix} |0\rangle\ , \tag{2.7}$$

where $\vec{n} = (n_1, \ldots, n_N)$ and we have chosen $n_1 > n_2 > \ldots > n_N \geq 0$. Note that $|f\rangle$ corresponds to the minimal occupation configuration, determining the Fermi sea level and defined by $\vec{n}_f = (N-1, N-2, \ldots, 1, 0)$.

At this point, a general state can be represented by the occupation vector $\vec{n}$, that in turn has the associated U(N) Young diagram



This diagram tells us that we should take the creation operators $a_i^\dagger$ to the n_i-th power and completely antisymmetrized their action on the vacuum $|0\rangle$. In particular, we can relabel the states such that we count the excitations above the Fermi sea level, by defining a new vector $\vec{n}' = \vec{n} - \vec{n}_f$. Keeping only the non vanishing n_i', we get smaller Young diagrams (less rows), telling us only about the excitations *above the Fermi sea level.* Note that, in our conventions, the operator located at the Fermi sea level is $a_1^\dagger$ while the one at the bottom corresponds to $a_N^\dagger$. For example, the following Young diagram



corresponds to exciting only the first two oscillators to the n_1' and n_2' energy levels respectively, over the Fermi sea.[9]

As we have seen, this reduced system comes naturally in terms of fermionic creation and annihilation operators acting on the vacuum. GGs were defined in (2.1) in terms of the

[9]Therefore, energy and angular momentum (R-charge) will be measured from the Fermi sea level E_f.

complex scalars Φ, and not of the operators A and B. Nevertheless, in the BPS sector of the Hilbert space we are considering, $A^\dagger$ is proportional to $\Phi^\dagger$ up to normalization factors (just note that B annihilates any state that we have considered, and then use the definition of $A^\dagger$ and B to obtain the above result). Therefore, in this framework, we can describe equivalently GGs by considering Schur polynomials in $A^\dagger$.

Fortunately, Schur polynomials have already been studied in depth, and many of their properties are under control. In particular, Schur polynomials have a very convenient decomposition in terms of Slater determinants (see for example [20]) given by

$$\chi^\dagger_{n,R} = \frac{\det((a_j^\dagger)^{\hat{n}_i+N-i})}{\prod_{k<l}(a^\dagger{}_k - a^\dagger{}_l)}\,, \tag{2.8}$$

where the term in the numerator is the determinant of a $N\times N$ matrix labeled by (i,j), $\hat{n}_i$ is the partition of n (i.e. $n = \sum \hat{n}_i$) defined by the specific representation R of the symmetric group S_n used ($\hat{n}_i$ is the number of boxes in the i-th row of the Young tableau associated to the representation R of S_n). In particular, if we are considering GGs in AdS$_5$, we are instructed to use the U (or totally symmetric) representation, that has the following partition

$$\hat{n}_i = n\delta_{i,1} \qquad i = 1,\ldots,N\,, \tag{2.9}$$

but if we consider GG in $\mathcal{S}^5$, we use the U' representation, with partition

$$\hat{n}_i = 1 \qquad i = 1,\ldots,N\,. \tag{2.10}$$

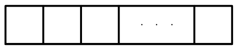
Therefore, a normalized operator representing a GG in AdS acting on the fermionic vacuum $|f\rangle$ gives

$$\chi^\dagger_{(n,U)}\,|f\rangle = \frac{1}{\sqrt{f_{ns}f_v}}\det\begin{pmatrix} (a_1^\dagger)^{n+N-1} & (a_1^\dagger)^{N-2} & \cdots & (a_1^\dagger)^0 \\ (a_2^\dagger)^{n+N-1} & (a_2^\dagger)^{N-2} & \cdots & (a_2^\dagger)^0 \\ \vdots & \vdots & \ddots & \vdots \\ (a_N^\dagger)^{n+N-1} & (a_N^\dagger)^{N-2} & \cdots & (a_N^\dagger)^0 \end{pmatrix}|0\rangle\ , \tag{2.11}$$

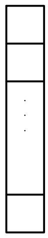
where $f_{ns} = (n+N-1)!/(N-1)!$. Note that, only the first column has an exponent different from the assigned value characteristic of the Fermi sea. On the other hand, a normalized GG in $\mathcal{S}^5$ is given by the expression

$$\begin{aligned}\chi^\dagger_{(n,U')}\,|f\rangle &= \frac{1}{\sqrt{f_{na}f_v}}\times \\ &\times\det\begin{pmatrix} (a_1^\dagger)^N & (a_1^\dagger)^{N-1} & \cdots & (a_1^\dagger)^{N-n+1} & (a_1^\dagger)^{N-n-1} & \cdots & (a_1^\dagger)^0 \\ (a_2^\dagger)^N & (a_2^\dagger)^{N-1} & \cdots & (a_2^\dagger)^{N-n+1} & (a_2^\dagger)^{N-n-1} & \cdots & (a_2^\dagger)^0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ (a_N^\dagger)^N & (a_N^\dagger)^{N-1} & \cdots & (a_N^\dagger)^{N-n+1} & (a_N^\dagger)^{N-n-1} & \cdots & (a_N^\dagger)^0 \end{pmatrix}|0\rangle\ ,\end{aligned} \tag{2.12}$$

where $f_{na} = (N-n+1)!/(N-1)!$. Note this time, that there is a jump in the value of the exponent in the column number $n+1$. Also, the above states form an orthonormal basis for GGs in AdS, i.e. $\langle f|\,\chi_{(n,U)}\chi^\dagger{}_{(m,U)}\,|f\rangle = \delta_{nm}$, and similarly for GGs in $\mathcal{S}^5$. Using the rules given before (regarding how to associate a $\mathrm{U}(N)$ Young diagram to a given state), both GGs have very simple $\mathrm{U}(N)$ diagrams, given respectively as



GG in AdS_5



GG in $\mathcal{S}^5$

where the first diagram is telling us that GGs in AdS_5 correspond to exciting the operator $a_1^\dagger$ (up to the $\mathbb{Z}_N$ action) high above the Fermi sea level with multiplicity n. Also, it is important to see that, due to the form of the diagram, there is no bound on how much we can excite this operator, a characteristic feature of GGs in AdS_5. Instead, the behavior of the second diagram is completely different, and indeed it has a maximum value for n given by $n = N$. This value is exactly the depth of the Fermi sea, and this time the interpretation of the diagram is that a GG in $\mathcal{S}^5$ corresponds to uplift the Fermi sea level by one unit, creating a hole deep down into it [14].

2.2 Quasi-classical states

In the above framework, it is useful to remember that each $a_i^\dagger$ is related to the position of one of the corresponding D3-branes that form the $\mathrm{U}(N)$ theory. Therefore, GGs in AdS can be understood as the result of separating one D3-brane from the stack of branes that produces the $\mathrm{AdS}_5 \times \mathcal{S}^5$ geometry. This point of view was already foreseen in [8], where a semi-classical treatment of the action (2.3) was used. Actually, this semi-classical picture seems to capture a lot of the physics, giving the correct BPS bound, energy and R-charges of the system, suggesting that we are really facing a quantum system that somehow behaves classically. This type of phenomenon has already occurred in the AdS/CFT duality, when large quantum numbers are involved (see for example [21]).

To analyse this behavior in the matrix model under consideration, we shall use *coherent states*, defined by the requirement that they minimize the uncertainty principle. Asking that the quantum uncertainty in the energy is much smaller than the measure of the energy itself, we obtain *quasi-classical states*. This further condition corresponds to consider a large R-charge limit, with the energy well approximated by the classical value. More precisely, in appendix A, we show that a quasi-classical state $|\alpha\rangle$ can be defined by

$$\mathrm{Tr}(\hat{A})\,|\alpha\rangle = \sqrt{k}\,\alpha\,|\alpha\rangle\ , \qquad \frac{\Delta E}{E} \ll 1\,, \tag{2.13}$$

where $E = \langle\alpha|\,H\,|\alpha\rangle - E_f$ is the expectation value of the energy above the Fermi surface and $\Delta E^2 = \langle\alpha|\,H^2\,|\alpha\rangle - \langle\alpha|\,H\,|\alpha\rangle^2$ measures its uncertainity. The integer $k \leq N$ defines the number of D3-branes which are separated from the initial stack of N D3-branes to build the GGs, if the classical energy is of the form $E_{\mathrm{classical}} = \omega\alpha^*\alpha$.

Indeed, we have observed that quasi-classical states of GGs in AdS_5 can be constructed, providing a link between the classical solution of [8] and our quantum matrix model. Here we show the resulting states and the main properties, but a detailed discussion can be found in appendix A.

In particular, this set of equations can be solved in the case $k = 1$, describing a single GG in AdS. The resulting coherent state is defined by (A.11),

$$|\alpha\rangle = \frac{1}{\sqrt{f_\alpha}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{(N+n-1)!}} \chi^\dagger_{(n,U)} |f\rangle , \qquad \text{with} \qquad f_\alpha = \sum_{n=0}^{\infty} \frac{(\alpha^* \alpha)^n}{(N+n-1)!} . \tag{2.14}$$

This state is explicitly constructed as a superposition of Schur polynomials in the U representation, which represent a GG in AdS. Hence, as stated, $|\alpha\rangle$ is a coherent GG in AdS. As we will show in the next section, this coherent state corresponds to have a quasi-classical state where only one creation operator $a_i^\dagger$ (i.e. only one brane) has been excited above its ground state, reproducing exactly the classical solution of [8].

The second coherent state we achieved to construct is for $k = N$. In this case, all the N D3-branes are excited with the same angular momentum, and are described in the CFT by the following state,

$$|\Omega\rangle = \frac{1}{\sqrt{f_N}} \exp\left(\frac{\alpha}{\sqrt{N}} \, \mathrm{Tr}\, \hat{A}^\dagger \right) |f\rangle , \qquad \text{with} \qquad f_N = \exp\left(-\alpha^* \alpha\right) . \tag{2.15}$$

Finally, we showed that it is not possible to construct a coherent state describing a single GG in $\mathcal{S}^5$ or, in other words, as linear combination of Schur polynomials in the U' representation acting on the Fermi vacuum. This means that such GGs are quantum in nature, and appear delocalized in space-time from the ten dimensional point of view. In particular, their energy is not well defined classically, and hence the supergravity approximation will fail in the bulk.

3. Back reaction of giant gravitons

In the previous section we described GGs from the point of view of the CFT theory dual to $AdS_5 \times \mathcal{S}^5$ space-time, in particular we have obtained quasi-classical states corresponding to GGs in AdS by means of a coherent superposition of quantum states. On the gravitational side of the correspondence, this limit can be interpreted as the localization of the branes in spacetime, and a description in terms of supergravity fields makes sense in the bulk. Since these extended objects act as sources for both the gravitational field and the Ramond-Ramond five-form, their presence should deform the $AdS_5 \times \mathcal{S}^5$ space-time on which they live. The purpose of this section is to deduce the form of this back reaction.

To this end, we evaluate the expectation value of the stress energy tensor of the CFT theory. Then, using the AdS/CFT correspondence, we translate it into the boundary stress energy tensor of the supergravity solution sourced by these GGs, and reconstruct the bulk fields. Finally, we discuss the validity of this solution and examine the case of quasi-classical states.

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3.1 General analysis

First of all, on the CFT side, the configuration we are considering does not involve gauge fields, as the D3-branes are not sources for the dilaton and the axion. Moreover, the energy of the GG does not depend on $g_{\rm YM}$ and therefore the commutator term of the SYM should not play any role. The relevant part of the CFT action on the boundary $\mathbb{R} \times \mathcal{S}^3$ is,

$$S = -\frac{1}{2g_{\rm YM}^2} \int d^4x \sqrt{-h} \sum_{m=1}^{6} \mathrm{Tr} \left(\partial_a X^m \partial^a X^m + \frac{1}{\ell^2} (X^m)^2 \right) \tag{3.1}$$

where the $1/\ell^2$ term is due to the coupling with the background curvature, imposed by the conformal invariance. When restricted to homogeneous configurations this action reduces to three copies of the model (2.3) studied in the previous section. The associated stress energy tensor T_{ab} is given by

$$\begin{aligned} T_{ab} = \frac{2}{3g_{\rm YM}^2} \sum_{m=1}^{6} \mathrm{Tr} \Big[& \nabla_a X^m \nabla_b X^m - \frac{1}{2} X^m \nabla_a \nabla_b X^m - \\ & - \frac{1}{4} h_{ab} \left(\nabla_c X^m \nabla^c X^m - 2 X^m \Box X^m \right) + \frac{1}{4} G_{ab} X^m X^m \Big] \end{aligned} \tag{3.2}$$

where G_{ab} is the Einstein tensor of the background manifold $\mathbb{R} \times \mathcal{S}^3$, and h_{ab} is its metric. If we introduce the unit time vector $v^a = (1,0,0,0)$, this tensor reads

$$G_{ab} = -\frac{1}{\ell^2} \left(h_{ab} - 2 v_a v_b \right) . \tag{3.3}$$

Then, assuming that the scalar fields do not depend on the coordinates on the three-sphere and using the equations of motion, we obtain the stress tensor

$$T_{ab} = \frac{2}{3g_{\rm YM}^2} \sum_{m=1}^{6} \mathrm{Tr} \left(\dot{X}^m \dot{X}^m + \frac{1}{\ell^2} X^m X^m \right) \Theta_{ab} \,, \tag{3.4}$$

where we have defined

$$\Theta_{ab} \equiv v_a v_b + \frac{1}{4} h_{ab} \,. \tag{3.5}$$

This stress tensor comes in a perfect fluid form and in accordance with the conformal invariance of the theory, it is traceless. At this point, it is important to notice that $T_{\mu\nu}$ is proportional to the hamiltonian (2.4) of the reduced theory. Hence, on the quantum theory the expectation value of the stress energy tensor in a general state $|\psi\rangle$ gives

$$\langle T_{ab} \rangle = \frac{4}{3 \Omega_3 \ell^3} \langle \psi | \, H \, | \psi \rangle \, \Theta_{ab} \,. \tag{3.6}$$

In the full SYM theory, we have here an additional contribution due to the Casimir effect, which is however known to match the vacuum contribution of the supergravity theory. By neglecting this term, we are simply describing the excitations above the vacuum of the theory.

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In general,

$$H = \frac{1}{\ell} \sum_{I=1}^{3} \left[J_I + 2\,\mathrm{Tr}\left(B_I^\dagger B_I \right) \right], \tag{3.7}$$

where J_I are the R-charge operators and the trace counts the total occupation number of the B_I oscillators. We are interested in BPS states, where only $A_I^\dagger$ excitations are present. Therefore this trace term does not contribute to the total energy of the system and the expectation value of the stress energy tensor is

$$\langle T_{ab} \rangle_{\mathrm{BPS}} = \frac{4J}{3\Omega_3 \ell^4} \Theta_{ab}\,, \tag{3.8}$$

where $J = \langle \sum_I J_I \rangle$ is the expectation value of the total R-charge. Note that the total energy of the state is

$$E = \langle T_{tt} \rangle\, \Omega_3 \ell^3 = \frac{J}{\ell}\,, \tag{3.9}$$

which is the expected relation for BPS states.

The AdS/CFT conjecture tells us that this expectation value coincides with the boundary stress tensor of the corresponding supergravity solution. Hence, the back reaction of the giant graviton on the background is given, at least asymptotically, by the supergravity solution which has the correct charges, preserves the same amount of supersymmetries and whose boundary stress tensor is given by (3.6). One such solution is known: it is given by the supersymmetric Reissner-Nordström-AdS black hole studied in [15], and associated to a condensate of GG in $\mathcal{S}^5$ in [16], where this solution was named *superstar*.

The static R-charged black holes in AdS_5, solutions to the STU model, have a metric of the form [22]

$$ds^2 = -H(r)^{-2/3} f(r) dt^2 + H(r)^{1/3} \left(\frac{dr^2}{f(r)} + r^2 d\Omega_3^2 \right), \tag{3.10}$$

where the function $f(r)$ is given by

$$f(r) = 1 - \frac{m}{r^2} + \frac{r^2}{\ell^2} H(r) \tag{3.11}$$

and $H(r)$ is the product of three harmonic functions

$$H(r) = \prod_{I=1}^{3} H_I(r)\,, \qquad H_I = 1 + \frac{q_I}{r^2}\,. \tag{3.12}$$

Here, m is the non-extremality parameter (determining the mass of the solution) and the q_I are the R-charges in the $(I, I+3)$ planes of the five sphere. We parameterize the two scalars of the theory by three functions X^I constrained by the relation $X^1 X^2 X^3 = 1$. These functions, and the three U(1) gauge fields $\mathcal{A}_\mu^I$, read respectively

$$X^I = H^{\frac{1}{3}} H_I^{-1}\,, \qquad \mathcal{A}^I = \sqrt{1 + \frac{m}{q_I}} \left(1 - H_I^{-1} \right) \mathrm{d}t\,. \tag{3.13}$$

In the $m = 0$ limit, this solution preserves half of the supersymmetries [15], and a naked singularity develops. The associated boundary stress energy tensor is computed in appendix B, and assumes the form

$$\hat{T}_{\mu\nu} = \frac{1}{6\pi G_5 \ell^3} \left(Q + \frac{3\ell^2}{8} \right) \Theta_{\mu\nu} \,, \tag{3.14}$$

where Q is the total R-charge of the system ($Q = \sum_{I=1}^{3} q_I$). Here, G_5 is the five dimensional Newton's constant, related to the units of flux N of the five form and to the radius of AdS by

$$16\pi G_5 = \frac{4\Omega_3 \ell^3}{N^2} \,. \tag{3.15}$$

Expressed in terms of the CFT constants, the boundary stress tensor becomes

$$\hat{T}_{\mu\nu} = \frac{2N^2 Q}{3\Omega_3 \ell^6} \Theta_{\mu\nu} \,, \tag{3.16}$$

where we have dropped the Casimir contribution to the stress energy, since we are interested to the stress energy of the excitations above the vacuum. This result matches perfectly the stress energy tensor (3.8) obtained from the matrix model if we take the R-charges of the CFT state and those of the supergravity solution to be linked by the relation

$$\langle\psi| \, J_I \, |\psi\rangle = \frac{N^2}{2\ell^2} q_I \,. \tag{3.17}$$

Note that the charges and boundary stress tensor match also in the non-extremal case, with the same relation (3.17) between the charges, if the total B_I occupation number matches the non-extremality parameter m of the solution

$$\langle\psi| \sum_{I=1}^{3} \mathrm{Tr}\, B_I^\dagger B_I \, |\psi\rangle = \frac{3N^2}{8\ell^2} m \,. \tag{3.18}$$

In this case, the supergravity solution would describe a genuine black hole in AdS_5, but the validity of the matrix model description away from the supersymmetric state is not obvious and will be analyzed elsewhere [23].

3.2 Classical limit and supergravity description in ten dimensions

In the general analysis, we have been able to match the expectation value of the reduced CFT stress tensor with the boundary stress tensor of a family of supergravity solutions. Note that, in the matrix model, a generic quantum state $|\psi\rangle$ corresponds to excitations of the scalar fields X^m that typically describe a delocalized system of D3-branes. In particular, this is also true for the GG states constructed using single Schur polynomials. On the other hand, we can consider *quasi-classical* states, representing *localized GGs*, that certainly can be used as sources of supergravity.

For instance, we found in appendix A that GGs in AdS can be excited coherently. In the quasi-classical limit, these states allow to understand GGs in AdS as set of one or more D3-branes, separated from the initial stack of N D3-branes used to define the AdS/CFT duality. In particular, we have constructed explicitly quasi-classical states corresponding to a single GG in AdS and the state where all N D3-branes are excited simultaneously. In contrast, we showed that no analogous coherent states can be defined for GGs in $\mathcal{S}^5$, and consequently they have no supergravity description.

To be more specific, the state corresponding to a single quasi-classical GG in AdS is given by the coherent state $|\alpha\rangle$ defined by equation (2.14), with large $|\alpha|$.[10] In particular, we can take the state with initial conditions $\alpha_{10} = \sqrt{\frac{g_{\rm YM}^2 J_I}{\Omega_3 \ell^2}}$ and $\alpha_{i0} = 0$ for the other $i \geq 2$. Then, the scalar field expectation values behave as a classical solution of the theory, and take the values

$$\Phi^1 = \sqrt{\frac{g_{\rm YM}^2 J_I}{\Omega_3 \ell^2}}\hat{\eta} e^{it/\ell}, \tag{3.19}$$

where $\hat{\eta} = \mathrm{diag}(1, 0, \ldots, 0)$. Factoring out the U(1) factor of the U(N) gauge group, corresponding to the motion of the center of mass of the N D3-branes, $\hat{\eta}$ becomes the traceless diagonal $N \times N$ matrix

$$\hat{\eta} = \sqrt{\frac{N-1}{N}} \begin{pmatrix} \eta & & & \\ & -\frac{\eta}{N-1} & & \\ & & \ddots & \\ & & & -\frac{\eta}{N-1} \end{pmatrix}. \tag{3.20}$$

The solution given by equations (3.19) and (3.20) is exactly the classical CFT solution proposed in [8] as dual to a giant graviton in AdS$_5$.

The second system corresponds to the quasi-classical state with all α_i's having the same initial conditions (see equation (A.8)). In this case, we have a collective motion of all D3-branes, and the giant graviton condensate can be interpreted as a *stack of rotating D3-branes*. This solution, explicitly given in [24], corresponds to a type-IIB supergravity solution, which is asymptotically flat and consists in a black p3-brane displaced from the center and rotating in the transverse $\mathcal{S}^5$ directions. By carefully, and simultaneously, taking the near-horizon and extremal limits, one recovers the supersymmetric solution under consideration.

We would like to stress that these quasi-classical GGs in AdS cover all the possible ranges of energies that GGs can describe. In fact, we are able to obtain GGs with energies E, satisfying the inequality $E \geq \sqrt{N}$. This same inequality is obtained by demanding consistency in the test brane picture, where in particular, the equality corresponds to the case where we have GGs of almost Planck size [11]. For smaller energies, quantum effects become dominant.

As a last point, using the equation that links the radius of a test GG in AdS with its energy, $r/\ell = \sqrt{E\ell/N}$ [8], we get that its thickness goes to zero for large N. Hence, in the supergravity limit, the branes become infinitely thin, and the description of classical supergravity plus external Born-Infeld matter is justified. Obviously these are just qualitative

[10]It is well known that GGs are well defined as long as their R-charge goes at least like $\sqrt{N}$ [11]. We have checked that our quasi-classical states have a well defined classical limit even for the smaller GGs (see appendix A).

arguments, since the relation between the radius and the energy has been obtained in the test brane approximation. However, when $r \gg \ell_P$ we expect them to hold, and indeed they prove the result we already stated: *quasi-classical states of GGs in AdS are localized in space-time and their back reaction is well described by the supersymmetric RN-AdS solution in the bulk of* $AdS_5 \times \mathcal{S}^5$.

4. Summary and discussion

In this article we have developed a formalism in the $\mathcal{N} = 4$ SYM to deal with GGs in terms of a matrix model. This formalism only applies after a few simplifications have been made. Basically we have focused our attention to the scalar sector only and considered spatially homogeneous configurations. Also, we have set the string coupling to zero to obtain free fields. These approximations are justified for the states we are interested in, because they excite only the scalar sector, and the homogeneity corresponds to a freezing of the internal modes of these object. Moreover, since the states under consideration are BPS, we expect this description to remain valid as we turn on g_s. In this framework, GG creation operators are identified with completely symmetric or antisymmetric Schur polynomials acting on the Fermi sea describing the vacuum.

Translating the expectation value of the stress energy tensor into the boundary stress energy tensor of the corresponding supergravity solution, we were able to identify the back reaction on the AdS space-time due to the presence of these branes. The response of the supergravity fields turned out to be the BPS Reissner-Nordström-AdS solution, with one or more R-charges turned on. The resulting field configuration can be trusted in general only in the asymptotic region, since for a generic quantum state the giant gravitons are delocalized. Nevertheless, there are quasi-classical states for which the branes localize in space-time and, as long as the curvature of the spacetime remains small, the supergravity solution describes correctly the back reaction. These quasi-classical solutions are systems composed by one or more GGs in AdS. However, there is no analogous quasi-classical state for GGs in $\mathcal{S}^5$ and therefore the associated back reacted solution can only be trusted near the boundary. In computing the boundary stress tensor of the supergravity solution, we have also shown that the finite counterterm of Liu and Sabra [17] is needed to eliminate the scheme-dependent part of the conformal anomaly.

Regarding future avenues of research, we make notice that in the CFT matrix model, there is the possibility to discuss non-BPS excitations acting simultaneously with $A^\dagger$'s and $B^\dagger$'s on the vacuum. Presumably, if we remain in the near BPS regime, we can still keep control on the system, opening the way to the description of static near BPS AdS black holes. Also, the recent discovery of a BPS black hole in AdS opens up the possibility of an exact derivation of their entropy. As pointed out by Gutowski and Reall [25], these black holes could only be sourced by D3-branes, or GGs. We believe that, the extension of this matrix model by adding angular momentum in AdS, could provide the microscopic degrees of freedom of these objects. In particular, GG configurations carrying the correct quantum numbers (energy, angular momentum and R-charges) have been described in [5]. These issues are object of current research [23].

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A. Coherent and quasi-classical states

The matrices X^I of the CFT describe the positions of the D3-branes in spacetime, and for a general quantum state we expect them to be delocalized. Since we are interested in configurations that admit a supergravity description, we should look for states that behaves semi-classically.[11]

A standard approach to obtain such states is to use coherent states, which have the nice property that they saturate the uncertainity principle.

A.1 Coherent states

In the matrix model, due to the $\mathrm{U}(N)$ invariance, single components of our matrices are not observable. Nevertheless, the N *eigenvalues* of X^m are gauge invariant, and describe the positions of the D3-branes (here, the residual $\mathbb{Z}_N$ symmetry corresponds to the fact that the N branes cannot be distinguished among them).

A particularly interesting gauge-invariant observable is given by the center of mass of the branes, defined by

$$Y^m = \frac{1}{N} \operatorname{Tr} X^m \,. \tag{A.1}$$

In particular, this observable carries all the physical information, for the simple case where a group of n branes are displaced all together out of the original stack of N D-branes. In this case, we have that for X^m, n eigenvalues are equal to λ and $N-n$ are equal to zero.

The conjugate operators are then given by the total momenta

$$P_m = \operatorname{Tr} \pi_m \,, \qquad \pi_m = \frac{\partial \mathcal{L}}{\partial \dot{X}^m} \,, \tag{A.2}$$

and satisfy the canonical commutation relations $[Y^m, P_n] = \delta^m_n$. Using standard arguments, one observes that $\| \left(Y^m - Y^m_0 + i\lambda(P_m - P_{0m})\right) |\psi\rangle \|^2 \geq 0$ must hold for any λ and deduces that the uncertainty on Y^m and P_m is bounded by

$$\Delta Y^m \Delta P_m \geq \frac{1}{2} \,, \tag{A.3}$$

in units where $\hbar = 1$. To achieve minimal uncertainty, we have to consider states which saturate this inequality and therefore satisfy $(Y^m + i\lambda P_m)\,|\tilde{\alpha}\rangle = \tilde{\alpha}^m\,|\tilde{\alpha}\rangle$ for some λ. Here, we have labeled the state with a set of complex numbers, determined by the initial conditions $\tilde{\alpha}^m_0 = Y^m_0 + i\lambda P_{0m}$. In addition, we request that $Y^m + i\lambda P_m$ annihilates the vacuum

[11] In this article, we work exclusively in the Heisenberg picture.

of the theory. This condition fixes $\lambda = 1/N\beta^2$. The resulting states are called *coherent states*, and their defining relation is given by

$$\left(\beta N Y^m + \frac{i}{\beta} P_m\right) |\alpha\rangle = \alpha^m |\alpha\rangle \,. \tag{A.4}$$

Let us consider now BPS states. These are defined by the relation $B^I |\psi\rangle = 0$, which reads in term of the original variables

$$\left(\beta X^I + \frac{i}{\beta}\pi_I\right) |\psi\rangle = i\left(\beta X^{I+3} + \frac{i}{\beta}\pi_{I+3}\right) |\psi\rangle \ . \tag{A.5}$$

In other words, there is must be a phase shift of $\pi/2$ between the oscillators X^I and X^{I+3}. From this relation, it follows that

$$\operatorname{Tr} A^I |\psi\rangle = \left(\beta N Y^I + \frac{i}{\beta} P_I\right) |\psi\rangle = i\left(\beta N Y^{I+3} + \frac{i}{\beta} P_{I+3}\right) |\psi\rangle \,. \tag{A.6}$$

Therefore, a BPS coherent state $|\alpha\rangle$ in Y^m, defined by (A.4) with $\alpha^I = i\alpha^{I+3}$, can be equivalently characterized by the relation

$$\operatorname{Tr} A^I |\alpha\rangle = \alpha^I |\alpha\rangle \ , \qquad \alpha^I \in \mathbb{C}, \qquad I = 1\ldots 3\,. \tag{A.7}$$

We stress the fact that we have to construct $\mathrm{U}(N)$-invariant states, and therefore we cannot simply require the coherence in each element of the matrices X^m. The definition (A.7) is however gauge-invariant and defines a minimal uncertainty state, as required. Also it is nontrivial to find solutions to this equation, since we are working in the case where $A^1 = \hat{A}^1$ is diagonal, and the fermionic Hilbert space is obtained by acting with symmetric operators on the Fermi vacuum. Moreover, since we are mainly interested in condensates of D3-branes, the resulting coherent states should verify the appropriated symmetry properties.

A.2 Particular solutions to the coherent state equation

The simplest way to solve equation (A.7) is to act with an exponential of the operator $(\hat{A}^1)^\dagger$ on the vacuum of the theory, which is in our case the Fermi vacuum $|f\rangle$. Taking into account the phase difference for BPS states, this is equivalent to use an exponential of $\mathrm{Tr}(A^1)^\dagger$. Indeed, if we define

$$|\Omega\rangle = \frac{1}{\sqrt{f_N}} \exp\left(\frac{\alpha}{\sqrt{N}}\,\mathrm{Tr}(\hat{A}^1)^\dagger\right) |f\rangle\,, \qquad \text{with} \qquad f_N = \exp\left(-\alpha^*\alpha\right), \tag{A.8}$$

we easily obtain using the commutation relations $[\operatorname{Tr}\hat{A}^I, \operatorname{Tr}\hat{A}^\dagger_J] = N\delta^I_J$ that indeed this is a coherent state satisfying $\operatorname{Tr}\hat{A}^1 |\Omega\rangle = \sqrt{N}\alpha\,|\Omega\rangle$. Note that we have used a different normalization for α, without loosing generality (see equation (A.7)), that will be useful later. Moreover, this state is gauge-invariant, as it should be, since it corresponds to a symmetric operator acting on the Fermi vacuum.

In general, a state satisfying (A.7) will be coherent in Y^I, however its composition in terms of branes and open strings excitations is not clear, and may be quite complicated due to the fermionic structure of the theory. To have a clear example of quasi-classical GG state, we will now build a coherent state $|\alpha\rangle$ describing the excitation of a single GG in AdS. To have this interpretation, the state must consist of a linear combination of Schur polynomials $\chi^{\dagger}_{n,U}$ acting on the Fermi vacuum. Therefore, we have to find complex coefficients γ_n such that the following two equations hold,

$$\mathrm{Tr}\,\hat{A}^1\,|\alpha\rangle = \alpha\,|\alpha\rangle\ , \qquad |\alpha\rangle = \sum_{n=1}^{\infty}\gamma_n\chi^{\dagger}_{n,\,U}\,|f\rangle\ . \tag{A.9}$$

A key relation to solve equation (A.9), and to obtain an explicit form of the coherent state $|\alpha\rangle$ is

$$\mathrm{Tr}(\hat{A}_1)\,\chi^{\dagger}_{(n,U)}\,|f\rangle = \sqrt{(n+N-1)}\,\chi^{\dagger}_{(n-1,U)}\,|f\rangle\ . \tag{A.10}$$

To prove this formula, one has to rewrite the ket $\chi^{\dagger}_{(n-1,U)}\,|f\rangle$ in terms of its occupation numbers as $|n+N-1,N-2,\ldots,1,0\rangle_A$, where the subscript A corresponds to antisymmetrization. Then, every time that one of the a_i hits a term of the expansion in the antisymmetrization of the above ket, we get a term that already existed on the expansion (unless it contains the factor $(a^{\dagger})_i^{n+N-1}$) that, due to the antisymmetrization, will give zero. Therefore, only the terms with exponent $n+N-1$ will contribute, producing this number as an overall factor. These surviving terms have one less operator $a^{\dagger}_i$, since we have acted on them with a_i, and hence produce a ket of the form $|n+N-2,N-2,\ldots,1,0\rangle_A$ i.e. a Schur polynomial of degree $n-1$, $\chi^{\dagger}_{(n-1,U)}$ acting on the Fermi sea. The form of the final overall value $\sqrt{n+N-1}$ is the result of the ratio of the difference in the normalization between $\chi^{\dagger}_{(n,U)}$ and $\chi^{\dagger}_{(n-1,U)}$.

At this point, to obtain the final form of the coherent state, we just have to use the equation (A.10) into equation (A.9), getting a recursive series in the α_n. Then, the overall undetermined constant is fixed by the usual normalization requirement. The explicit form of the state is given by

$$|\alpha\rangle = \frac{1}{\sqrt{f}_\alpha}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{(N+n-1)!}}\,\chi^{\dagger}_{(n,U)}\,|f\rangle\,, \qquad \text{with} \qquad f_\alpha = \sum_{n=0}^{\infty}\frac{(\alpha^*\alpha)^n}{(N+n-1)!}\ , \tag{A.11}$$

where the normalization factor f_α can also be written as,

$$f_\alpha = \frac{1}{(\alpha^*\alpha)^{N-1}}\left(e^{\alpha^*\alpha} - \sum_{n=0}^{N-2}\frac{(\alpha^*\alpha)^n}{n!}\right)\ . \tag{A.12}$$

The above procedure can not be implemented exactly for GGs in $\mathcal{S}^5$. Basically, the problem is related to the fact that we only count with a finite number of such independent states, since there are only N independent Schur polynomials in the U' representation. To see how the obstruction arises, we first try to solve the analogous equation of (A.9),

$$\mathrm{Tr}\,\hat{A}_1\,|\beta\rangle = \beta\,|\beta\rangle\ , \tag{A.13}$$

$$|\beta\rangle = \sum_{n=1}^{N}\rho_n\chi^{\dagger}_{(n,\,U')}\,|f\rangle\ . \tag{A.14}$$

These equations can be solved by means of the following relation among normalized GGs in $\mathcal{S}^5$,

$$\mathrm{Tr}(\hat{A}_1)\, \chi^{\dagger}_{(n,U')} \left|f\right\rangle = \sqrt{N-n+1}\, \chi^{\dagger}_{(n-1,U')} \left|f\right\rangle \ . \tag{A.15}$$

which in turns, is simple to proof with reasoning parallel to the one used to proof (A.10). After substituting in (A.13), we get the relations

$$\rho_n = \frac{(N-n)!}{N!}(\beta)^n \rho_0 \qquad \text{for } n = 0, \ldots, N\,, \tag{A.16}$$

$$\rho_N = 0\,, \tag{A.17}$$

that have as only solution $\rho_n = 0$ for any n. *Therefore it is not possible to construct a coherent state made as a superposition of GGs in $\mathcal{S}^5$.*

Nevertheless, we point out that since we work within the AdS/CFT correspondence, N is considered to be large. We can therefore think on a combination of GGs in $\mathcal{S}^5$ that almost satisfy our constraints, defining an approximated coherent state. For example, we could arbitrary forget about equation (A.17), keeping all the others ρ_n. This is certainly an option, but the resulting ket will spread out in time for any finite N, and the classical limit will loose the very important requirement of localization. The D3-branes will delocalized after a given scale of time, and from the ten-dimensional point of view, no supergravity solution will describe the interior of the corresponding spacetime.

A.3 Quasi-classical states

Previously, we have considered the definition of coherent states in our model. Nevertheless, we can always impose the extra condition that the quantum uncertainty in the measurement of the energy ΔE, is much smaller than the expectation value of the energy E. In this way, our coherent states reproduce more closely the behavior of the classical observables. In this work, we will call quasi-classical states, to any coherent states that satisfy this extra condition.

We define our quasi-classical state by means of the following equations:

$$\mathrm{Tr}(\hat{A}) \left|\alpha\right\rangle = \sqrt{k}\,\alpha \left|\alpha\right\rangle \ , \qquad \frac{\Delta E}{E} \ll 1\,, \tag{A.18}$$

where $E = \left\langle\alpha\right| H \left|\alpha\right\rangle - E_f$ is the expectation value of the energy above the Fermi surface and $\Delta E^2 = \left\langle\alpha\right| H^2 \left|\alpha\right\rangle - \left\langle\alpha\right| H \left|\alpha\right\rangle^2$ measures its uncertainity. Note that the first equation, corresponding to the coherent state equation, has been modified by a factor of $\sqrt{k}$, to accommodate the possibility of describing k excited semi-classical branes. This will be clarified at the end of the appendix when we discuss the form of the classical solutions of the theory.

Armed with the above definitions, it is not difficult to see that the coherent state of equation (A.8) is also a quasi-classical state for the case $k = N$ and therefore is identified with the case were all the D3-branes have a total angular momentum in the plane defined by (X^1, X^4). In this case, is not difficult to calculate that

$$E = \omega \left|\alpha\right|^2 , \qquad \text{and} \qquad \frac{\Delta E}{E} = \frac{1}{\left|\alpha\right|}\,, \tag{A.19}$$

which in the limit of large $|\alpha|$ and large N, gives the desired result.

Also, the coherent state of equation (A.11) is a quasi-classical state for the case $k = 1$, and therefore it must be identified with the classical solution of a single GG in AdS of [8].

To verify the above statement, we first obtain the form of the energy,

$$E = \frac{\omega}{f_\alpha} \sum_{n=0}^{\infty} \frac{n\,(\alpha^*\alpha)^n}{\sqrt{(N+n-1)!}}\,, \tag{A.20}$$

which can be rewritten as

$$E = \omega|\alpha|^2 - \frac{\omega\gamma(N,|\alpha|^2)}{\gamma(N-1,|\alpha|^2)} \tag{A.21}$$

using the incomplete gamma function. Then, after some calculations, we obtain that

$$\frac{\Delta E}{E} = \left(\frac{|\alpha|^2}{E^2} - \frac{|\alpha|^2 \exp\left(|\alpha|^2\right)}{|\alpha|^2\,\gamma(N-1,|\alpha|^2) - \gamma(N,|\alpha|^2)} \right)^{1/2}. \tag{A.22}$$

Let us consider now two different large $|\alpha|$ limits, that define quasi-classical regimes. The first one corresponds to take $|\alpha|^2 \gg N$, that gives

$$E = \omega|\alpha|^2 + \mathcal{O}\left(\frac{1}{|\alpha|^2}\right), \qquad \text{and} \qquad \frac{\Delta E}{E} = \frac{1}{|\alpha|^2} + \mathcal{O}\left(\frac{1}{|\alpha|^3}\right). \tag{A.23}$$

A second asymptotic expansion can be obtained by taking $|\alpha|^2 = N + \sqrt{2N}y$, where $y \in \mathbb{R}$ is bounded and N large. Here we found that

$$\begin{aligned} E &= \omega\sqrt{2N}\,[y + g(y)] + \mathcal{O}\left(N^0\right), \\ \frac{\Delta E^2}{E^2} &= \frac{1}{2\,(y+g(y))^2} - \frac{g(y)}{y+g(y)} + \mathcal{O}\left(\frac{1}{\sqrt{N}}\right). \end{aligned} \tag{A.24}$$

The function $g(y)$ is defined by

$$g(y) = \frac{e^{-y^2}}{\sqrt{\pi}\,\mathrm{erfc}(-y)}\,, \tag{A.25}$$

and vanishes very rapidly when y is positive. Hence, the above expressions can be approximated for $y \gtrsim 1$ by

$$E \approx \omega\,\sqrt{2N}\,y\,, \qquad \text{and} \qquad \frac{\Delta E}{E} \approx \frac{1}{\sqrt{2y}}\,. \tag{A.26}$$

Remember that, in the test brane picture, the minimal energy that can be reached is precisely of order $\sqrt{N}$ [11]. Therefore, putting together the two limiting cases described above we cover all the possible energy ranges of a GG in AdS that can be described in the test brane picture, from the smallest $E \sim \sqrt{N}/\ell$ GGs to the cosmological ones.

As a last point, we review the form of the classical solution, to justify the interpretation we have made on the number of classical GGs in a given quasi-classical state. In the classical theory, the relevant part of the hamiltonian is of the form $H_{\text{classical}} = \omega\,\mathrm{Tr}\,(\alpha^*\alpha)$ with $\alpha = \frac{1}{\sqrt{2}}\left(\beta\Phi + \frac{i}{\beta}\Pi^*\right)$ and $\Pi_I = \frac{\partial\mathcal{L}}{\partial\dot{\Phi}^I}$. After having diagonalized the matrix α, the classical equations of motion for the diagonal elements α_i have the general solution $\alpha_i(t) = \alpha_{i0}e^{-i\omega t}$, with energy $E = \omega\sum_{i=1}^{N}\alpha_{i0}^*\alpha_{i0}$. Here, the coefficients α_{i0} encode the information of the initial conditions.

Following the interpretation given in [8] of the above classical system, consider the case where k oscillators have the same initial condition $\alpha_{i0} = \sqrt{k}\alpha_0$ for $i = 1, \ldots, k$ and all others are zero. This system corresponds to have k GGs with the same energy. If we evaluate the hamiltonian H, and the operator $\mathrm{Tr}\,\alpha$, in this configuration, we get $H_{\text{classical}} = \omega\, \alpha_0^* \alpha_0$, and $\mathrm{Tr}\,\alpha = \sqrt{k}\alpha_0 e^{-iwt}$. Note that, the above relation characterizes the classical state of k GGs in AdS. In particular in the $k = 1$ case, we are in the presence of a single GG, while for $k = N$, we are in the case where all the branes have the same R-charge. This last two extreme cases correspond to different identifications of the U(1) in U(N).

In the quantum system, the hamiltonian is again the sum of N harmonic oscillators, but we have to consider only gauge invariant observables, antisymmetrized in the N oscillators variables. Equation (A.7) gives our generalized definition of coherent state. The operator $\mathrm{Tr}\,\hat{A}^1$ has an equation of motion of the same form as the single operators a_i's, with general solution $\mathrm{Tr}\,\hat{A}^1 = (\mathrm{Tr}\,\hat{A}^1)_0 e^{-i\omega t}$. At this point $(\mathrm{Tr}\,\hat{A}^1)_0$ is a complex number defining the initial condition (note that we have restricted only the total sum of the initial condition of each operator a_i, and therefore many different combinations of initial conditions in a_i will have the same total initial condition in $\mathrm{Tr}\,\hat{A}^1$ in the same manner as for the classical theory).

Therefore, the use of equation (A.18) guaranties that the corresponding quasi-classical state has the classical interpretation of k displaced branes.

B. Boundary stress tensor

In this appendix we compute the boundary stress energy tensor for the Reissner-Nordström-AdS_5 family of black holes with three independent charges in the STU model. It is defined by the quasi-local stress tensor of Brown and York [26], as the variation of the gravitational action S_0 (including the Gibbons Hawking surface term to have a well-defined variational principle [27]) with respect to the metric $g_{\mu\nu}$ induced on the boundary $\partial\mathcal{M}$ of the manifold,

$$T^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_0}{\delta g_{\mu\nu}}\,. \tag{B.1}$$

The action, and the stress tensor, are regularized by taking the boundary at finite radius r. As $\partial\mathcal{M}$ is pushed to infinity, the stress tensor diverges. In the framework of AdS/CFT it is natural to renormalize the supergravity action by the addition of local counterterms on the boundary [28]. This method has the advantage of being background independent and covariant. In AdS_5, the needed counterterm has the form [28, 29]

$$S_{ct}[g_{\mu\nu}] = \frac{1}{8\pi G_5} \int_{\mathcal{M}} \sqrt{-g} \left[-\frac{3}{\ell} \left(1 - \frac{\ell^2}{12}\mathcal{R} \right) \right], \tag{B.2}$$

where $\mathcal{R}$ is the curvature scalar of the boundary metric. There is still the freedom to add *finite counterterms*, which are needed if one wants to recover for example a consistent thermodynamics for these black holes [30]. In [17], inclusion of such counterterms was advocated in order to recover the expected ADM energies and a BPS-like linear relation between energy and R-charges. This corresponds to a shift in the renormalization prescription in the dual CFT.

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What we show in this appendix, is that the deep reason for the introduction of this term is to *eliminate the scheme-dependent part of the Weyl anomaly.* Hence, by requiring a regularization procedure which respects the full conformal symmetry, we are able to lift the ambiguity in the definition of the action and conserved charges of the theory.

For the three-charges Reissner-Nordström-AdS$_5$ solutions (3.10), the quasi-local stress tensor obtained from the action $S_0 + S_{ct}$ reads

$$8\pi G_5 T_{\mu\nu} = \frac{4}{3\ell r^2}\left(\frac{3\ell^2}{8} + \frac{3}{2}m + Q\right)\Theta_{\mu\nu} - \frac{1}{\ell^3 r^2}\left(\tilde{Q} - \frac{1}{3}Q^2\right)g_{\mu\nu} + \mathcal{O}\left(\frac{1}{r^4}\right), \tag{B.3}$$

where $Q = \sum_I q^I$ is the total R-charge and $\tilde{Q} = \sum_{I<J} q^I q^J$, and $\Theta_{\mu\nu}$ is the tensor defined by (3.5). Note that there is a trace anomaly proportional to $\tilde{Q} - Q^2/3$, which originates from a choice of renormalization scheme which does not respect the asymptotic isometry group of AdS$_5$. From the CFT point of view, the regularization used violates the scale invariance. Indeed, the general form of the Weyl anomaly consists of a scheme independent part, which vanishes for the background under consideration, and a collection of total derivative terms that can be removed by adding suitable finite counterterms to the action [31, 32]. Hence, the anomaly we obtain is trivial, and it is possible to restore the conformal symmetry by adding to the action precisely the finite counterterm proposed by Liu and Sabra [17]. For the class of solutions (3.10) under consideration, its exact form is

$$S_{\phi^2} = \frac{1}{8\pi G_5}\int_{\mathcal{M}} \sqrt{-g}\left(\frac{1}{2\ell}h_{ij}\phi^i\phi^j\right), \tag{B.4}$$

where ϕ^i, $i = 1, 2$ parameterize the two scalars of the STU model, and h_{ij} is the moduli space metric. The contribution of the finite ϕ^2 counterterm is easily computed and is

$$8\pi G_5 T^{\phi^2}_{\mu\nu} = \frac{1}{\ell^3 r^2}\left(\tilde{Q} - \frac{1}{3}Q^2\right)g_{\mu\nu} + \mathcal{O}\left(\frac{1}{r^4}\right), \tag{B.5}$$

which is precisely the quantity needed to cancel the trace anomaly from the quasi-local stress tensor (B.3), and eliminate the nonlinear dependence on the charges. Finally, the complete quasi-local stress tensor reads,

$$8\pi G_5 T_{\mu\nu} = \frac{4}{3\ell r^2}\left(\frac{3\ell^2}{8} + \frac{3}{2}m + Q\right)\Theta_{\mu\nu} + \mathcal{O}\left(\frac{1}{r^4}\right). \tag{B.6}$$

To find the boundary stress tensor, we push the boundary $\partial\mathcal{M}$ to infinity, conformally rescaling the metric in such a way to eliminate the double pole. Defining the background metric upon which the dual field theory resides as

$$h_{\mu\nu} = \lim_{r\to\infty}\frac{\ell^2}{r^2}g_{\mu\nu} \tag{B.7}$$

we find that the boundary is the Einstein universe with metric $ds^2 = -dt^2 + \ell^2 d\Omega_3^2$. Hence, the CFT stress tensor is obtained by rescaling the quasi-local stress tensor by a factor r^2/ℓ^2 before taking the boundary at infinity. As a result, one obtains

$$8\pi G_5 T_{\mu\nu} = \frac{4}{3\ell^3}\left(\frac{3\ell^2}{8} + \frac{3}{2}m + Q\right)\Theta_{\mu\nu}\,. \tag{B.8}$$

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The AdS/CFT correspondence tells us that (B.8) coincides with the quantum expectation value of the CFT stress tensor:

$$\langle T_{\mu\nu}^{CFT} \rangle = T_{\mu\nu} \,. \tag{B.9}$$

The first term in the parenthesis of eq. (B.8) corresponds to the non-vanishing vacuum energy due to the Casimir effect of the CFT on $\mathbb{R} \times \mathcal{S}^3$, and the term proportional to $3m/2 + Q$ is the energy of the excitation over the vacuum.

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